REPORT DOCUMENTATION PAGE

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE 15 Dec 95	3. REPORT TYPE AND DATES COVERED Final Report: 1 June 92-30 Sept 95

4. TITLE AND SUBTITLE
Pulse Propagation in Random Media

5. FUNDING NUMBERS
F49620-92-J-0289
G

6. AUTHOR(S)
Werner E. Kohler

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Virginia Tech
Blacksburg, Virginia
24061

8. PERFORMING

AFOSR-TR-96
0093

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Dr. Arje Nachman
AFOSR/NM
110 Duncan Ave, Suite B115
Bolling AFB
Washington, DC 20332-0001

SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

12b. DISTRIBUTION CODE

19960320 058

13. ABSTRACT (Maximum 200 words)

Four studies involving the propagation of waves in random media were conducted. In the first of these, monochromatic elastic plane wave propagation within a randomly layered slab was considered. The issues of mode conversion upon reflection and localization were addressed. Mode conversion refers to the conversion of compressional to shear waves and vice versa by multiple scattering within the layering; localiation is the phenomenon wherein the random layering suppresses transmission of energy through the slab. Explicit characterizations of both phenomena were achieved. The second study addressed the issue of inversion. Radiation from an acoustic point source was assumed incident upon a randomly layered acoustic half space characterized by a deterministic refracting profile. The feasibility of reconstructing this refracting profile using the data recorded by multiple surface sensors from a single random realization was shown. The third study dealt with the issue of layering perturbations. A locally layered acoustic medium was considered, i.e. a medium in which random layering is subjected to controlled deformations or undulations. The basic question considered was the robustness of layering theory, i.e. do localization and other phenomena associated with random layering theory persist when the plane layers are perturbed? The results obtained establish that many of these properties are in fact preserved. The fourth study dealt with propagation loss in a single mode optical fiber. Of interest was the quantitative characterization of transmission loss, which arises from actual dissipation, localization and radiation loss.

14. SUBJECT TERMS			15. NUMBER OF PAGES 6
			16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT

Final Technical Report
Pulse Propagation in Random Media F49620-92-J-0289
92/06/01 - 95/09/30

The research conducted during this period was done in collaboration with George Papanicolaou (Stanford University), Benjamin White (Exxon Research & Engineering Co.) and Marie Postel (Laboratoire d'Analyse Numerique, UPMC et CNRS,Paris). In broad terms, the research focused upon direct and inverse problems involving wave propagation in complex heterogeneous media, modeled as random media. More specifically, the work has focussed upon the following areas:

(1) the multiple scattering of plane elastic waves within randomly layered elastic material. The manner in which the layering induces mode conversion (between shear and compressional waves) and localizes the waves (i.e. supppresses transmission) have been studied.

(2) the statistical inversion problem associated within the illumination of a randomly layered half-space with an acoustic point source. The regime considered is one in which the signal-to-noise ratio is extremely small. The return signal is essentially all "noise", i.e. multiply-scattered coda. Since the "noise", however, contains information about the substructure, the problem was to determine what information could be retrieved about the gross deterministic half-space substructure trends as well as the constitutive parameter fluctuation statistics.

(3) the propagation of acoustic signals in an acoustic layer in which the random sublayering has been warped or distorted in a controlled way. The study of propagation in such a "locally layered" medium was undertaken to determine the robustness of the layered theory. For example, is energy localized in such a medium?

(4) the study of propagation loss within an optical fiber. The manner in which the very small imperfections within a communication-grade fiber contribute to the (correspondingly small) propagation loss was quantified. Of particular interest in this analysis was the manner in which energy is coupled from a single propagating bound mode to the full spectrum of radiation modes. The transmission loss for the bound mode was found to arise from the following three sources:

(a) intrinisic dissipation, (b) localization effects due to bound mode-to-bound mode multiple scattering and (c) radiation loss

1. Localization and Mode Conversion in Randomly Layered Elastic Media:

We consider a randomly-layered lossless elastic slab illuminated from a homogeneous elastic half space by an obliquely incident monochromatic (CW) plane wave. The incident wave can be either a compressional wave or a shear wave. The half space below the slab is likewise assumed to be homogeneous in composition.The fluctuations of the constitutive parameters (Lame parameters) within the layered slab are not assumed to be small in amplitude; they are, however, assumed to be rapidly varying relative to wavelength. A small parameter, ε, is used to scale the problem. The slab width is taken to be O(1), the wavelength O(ε), and the layering correlation length O(ε^2) relative to this small parameter. When the incident wave penetrates the slab, it undergoes multiple scattering. In contrast to the analogous electromagnetic problem, the multiple scattering generates mode conversion (i.e. compressional to shear and vice versa) as well as change in propagation direction. As with other propagation problems in one-dimensional random media, the layering also localizes the wave. The transmission of elastic wave energy is supressed by the random layering; the transmission matrix norm decays exponentially as slab thickness is increased. A quantitative understanding of these two phenomena, mode conversion and localization, within an appropriate asymptotic limit wherein $\varepsilon \rightarrow 0$, was the goal of the study.

Explicit results were obtained for mode conversion upon reflection by a randomly layered half space. In this instance, then, we considered the limiting case where slab thickness becomes infinite. In this limit, all of the incident energy is ultimately reflected back into the homogeneous half space (because of localization). The question to be addressed, however, was the nature of this energy. For example, if a unit amplitude compressional wave was originally incident upon the layering, what fraction would be reflected as compressional energy and what fraction would be converted to reflected shear wave energy? An explicit expression was derived for the reflection law, i.e. the modulus of the 1,1 element (which equals that of the

2,2 element) of the reflection matrix. The reflection law was found to depend upon a single parameter which, in turn, depends upon four scattering correlation functions.

The second consideration addressed was that of localization and mode conversion upon transmission. In this instance, then, a finite thickness slab was considered. Explicit expressions were derived for the two Lyapunov exponents governing propagation through this slab (in the same asymptotic limit as small parameter $\varepsilon \to 0$). These exponents define two length scales that provide a (probabilty one) characterization of the transmission process. The larger Lyapunov exponent defines a shorter length, called the equilibration length, over which the relative strength of compression and shear energies settles into a fixed ratio. In other words, over an equilibration length, the transmission matrix collapses into a rank one matrix. The smaller Lyapunov exponent defines the longer localization length scale; on this longer scale the total transmitted elastic energy undergoes decay.

This work has been accepted for publication. It will appear as the following two-part publication: Localization and mode conversion for elastic waves in randomly layered media, Parts I and II, by W. Kohler, G. Papanicolaou and B. White, to appear in Wave Motion (in early 1996).

2. Statistical Inversion from Reflections of Spherical Waves by a Randomly Layered Medium:

In this study we consider radiation from an acoustic point source incident upon a randomly layered half space. In this case, the random zero mean layering is superposed upon a deterministic sound speed variation (i.e. a refracting profile) that is assumed to be unknown. The problem considered is the inverse problem, the task of retrieving information about the refracting profile and some general information about the noise (e.g. power spectrum information) using multiple measurements of the reflected pressure at the half space surface.

The nature of the probelm, then, is to consider a single realization of the random half space but to signal-process the reflected pressure at multiple receiver sites on the surface. What makes the problem particularly difficult is the fact that the reflected signal is virtually all "noise"; the saving feature, however, is the fact that this "noise" is not arbitrary. Rather, it is the result of multiple scattering of the signal by the random layering. As the signal undergoes this scattering and becomes randomized, it also encodes information about the deterministic background. Our analysis demonstrates the feasibility of this information extraction. By properly processing the reflected pressure at multiple surface receiver sites (from a single random realization illuminated by a single point source), one can reconstruct the deterministic sound speed profile at depth.

Point source, rather than plane wave, excitation makes the underlying hyperbolicity of the problem more difficult to exploit. The ray paths from the source to the various surface receivers are the end-result of refraction and multiple scattering by an a priori unknown sound speed. The algorithm used combines global optimization and layer peeling techniques. A piecewise-linear approximation to the underlying deterministic sound speed profile is ultimately generated by sequentially maximizing a partial objective function. The objective function uses the measured receiver power spectral data; the maximization is taken over a finite set of sound speed nodal values. Once the optimization at a given level is complete, the layer peeling aspect is achieved using the W-equations that were developed to study the direct problem (c.f. "Frequency Content of Randomly Scattered Signals", M. Asch, W. Kohler, G. Papanicolaou, M. Postel, B. White, SIAM Review, vol. 33, no. 4, 1991, p. 519-625).

This study has been accepted for publication. It will appear as " Statistical Inversion from Reflections of Spherical Waves by a Randomly Layered Medium", by M. Asch, W. Kohler, G. Papanicolaou, M. Postel and B. White, in the Journal of Computational Acoustics.

3. Wave Propagation in Locally-Layered Media:

The purpose of this study was to determine the robustness of layered medium theory. In other words, what happens to the reflection and transmission of acoustic waves when the layering is deformed? Are phenomena such as localization and O'Doherty-Anstey theory destroyed or robustly preserved when undulations are introduced into the layering?

The problem was studied in the same basic asymptotic regime as the prior two studies; a small parameter ε was introduced for scaling. Slab thickness is $O(1)$, wavelength is $O(\varepsilon)$ and correlation length is $O(\varepsilon^2)$. The layering undulations were introduced by defining :

$$z' = z + \varepsilon\phi(x,y,z)$$

where ϕ is a smooth O(1) function. For simplicity (to separate bulk from surface effects), we assume that the undulations "flatten out" at the slab boundaries so that the interfaces remain planar. Thus $\phi(x,y,0) = \phi(x,y,-L) = 0$. The perturbed layering was then introduced by assuming the reciprocal bulk modulus of the slab to have the form:

$$K^{-1} = K_1^{-1}[1 + \upsilon(z'/\varepsilon^2)] + \varepsilon K_{11}^{-1}(x,y,z)$$

Here K_1 is a positive constant, K_{11} is a deterministic O(1) perturbation and υ is a zero mean, stationary stochastic process bounded in modulus by a constant less than one. Thus the mean reciprocal bulk modulus is perturbed by an O(ε) spatially-varying component and the layering is caused to undulate with an amplitude comparable to wavelength but on a lateral scale large relative to wavelength.

The undulations are rather "gentle", therefore, and one would expect this to be a perturbed regime where many of the flat layering results might persist. The analysis, however, becomes considerably more complicated. On the one hand, the expressions for the transmitted and reflected pressures assume a natural and physically meaningful form. For an incident acoutic plane wave of the form:

$$p_{inc}(t,x,0) = \frac{1}{\sqrt{\varepsilon}}f(\frac{t - \kappa_0 \cdot x}{\varepsilon})$$

(where κ_0 is an incident slowness vector), the corresponding reflected pressure has the form:

$$p_{refl}(t,x,0) = \frac{-1}{2\pi\varepsilon}\int e^{i\omega(\kappa_0 \cdot x - t)/\varepsilon}[\iint \Gamma(\kappa_0,x,x',\omega,0)dx']\hat{f}(\omega)d\omega$$

The reflection coefficient Γ is a function of transverse vector-valued variables x and x' and the reflected pressure at x is thus obtained by integrating over the transverse planar surface.

The difficulty arises in solving for the reflection coefficient Γ. It now satisfies the following stochastic Riccati initial value problem:

$$\partial_z\Gamma + \frac{\varsigma}{\rho_1}\kappa \cdot (\nabla_x - \nabla_{x'})\Gamma = i\frac{\omega}{\varepsilon}n[2\Gamma - e^{-i2\frac{\omega}{\varepsilon}\tau}\delta(x - x') - e^{i2\frac{\omega}{\varepsilon}\tau}\iint\Gamma(\kappa,x,x'',\omega,z)\Gamma(\kappa,x'',x',\omega,z)dx'']$$

$$-i\frac{\rho_1}{\varsigma}[\phi_z(x,z) + \phi_z(x',z) + \frac{\varsigma}{\rho_1}\kappa \cdot (\nabla_x\phi(x,z) - \nabla_{x'}\phi(x',z))]\Gamma + i\frac{\omega}{2}\varsigma[K_{11}(x,z) + K_{11}(x',z)]\Gamma$$

$$\Gamma(\kappa,x,x',\omega,-L) = \Gamma_I(-L)e^{-i2\frac{\omega}{\varepsilon}\tau(-L)}\delta(x - x')$$

where x and x' are the transverse spatial coordinates, κ is slowness, τ is a travel time variable and ς and ρ_1 are derministic parameters. The function n is a zero mean noise term (essentially a deterministic multiple of υ). The task is to determine Γ and, in turn, use this to deduce statistical information about the reflected pressure p_{refl}.

We have attacked the problem by passing to the $\varepsilon \to 0$ asymptotic limit, obtaining a corresponding Ito version of the above stochastic Riccati equation. Using this approach, we have been able to deduce:
(I) the coherent reflected and transmitted pressures. The bulk modulus perturbation K_{11} is found to introduce a phase shift.
(II) the reflected and transmitted pressure intensities in the case of normal plane wave incidence. For this excitation, we deduce the robustness of the prior flat layering theory. The bulk modulus perturbation and the undulations are significant if we consider reflected pressure two-point correlations. However, if we collapse the spatial offset and study reflected pressure intensities, we recover the same W-equations arising in the flat layering theory.
(III) In the case of normal plane wave incidence, we likewise recover the localization results of flat layering theory. The transmission coefficient suffers exponential attenuation as slab thickness is increased.

(IV) O'Doherty-Anstey theory is found to persist in this perturbed random medium as well. Thus, if one makes a random coordinate transformation that synchronizes the observations of the leading pulse shape as it propagates through different realizations of this random medium, the shape is found to be deterministic in character.

Although these results are quite significant, the all-important issues of robustness of theory and localization, in particular, in the case of oblique incidence must still be resolved. A draft of a manuscript delineating our results to date has been written and is being prepared for publication.

4. Propagation Loss in an Optical Fiber:

Propagation loss in high quality communication optical fibers is quite small (roughly 0.35 dB/km.). This loss arises from a number of imperfections, such as intrinsic material dissipation, Rayleigh scattering due to material inhomogeneities formed at the time of solidification and geometric imperfections (e.g. microbends). The purpose of this study was to quantify this propagation loss; the aforementioned imperfections were modeled as very small amplitude random fields, which build up an O(1) effect over very long propagation paths (roughly 20 km.).

Parameter values were chosen which correspond to the propagation of a single bound mode and the continuum of radiation modes. A small parameter ε was introduced to quantify the scaling. The fiber core radius was chosen as the basic length scale. Relative to this length scale, wavelength and imperfection correlation length are O(1). The imperfection amplitudes were modeled as O(ε) and the propagation path length was taken to be O(ε^{-2}). For commercial grade fibers, a value of $\varepsilon \approx 10^{-5}$ is appropriate. Since the underlying theory is an asymptotic theory, valid in the limit $\varepsilon \to 0$, the asymptotic results should characterize the actual problem well.

The problem was analyzed using coupled mode theory, i.e. expanding the transverse electromagnetic fields in the modes of the unperturbed fiber. Introducing scattering variables and reflection and transmission matrices, one is led to the analysis of the following stochastic final value problem:

$$\frac{d}{dz}R = -RC_{11} + C_{22} + C_{21} - RC_{22}R, \quad R(L) = 0$$

$$\frac{d}{dz}T = -TC_{11} - TC_{12}R, \quad T(L) = I$$

Here R and T are generalized (discrete and continuous indexed) reflection and transmission matrices which describe the coupling of energy between the single bound mode and the radiation modes. The C_{ij} are zero-mean stochastic coupling blocks. (The scaling has, for simplicity, not been indicated.) Both propagating and evanescent radiation modes are included in this description. The analysis performed dealt with studying the asymptotic behavior of the appropriately scaled version of the above problem as $\varepsilon \to 0$. One can ultimately characterize quantities of interest, those that describe the bound mode, in terms of Markov diffusion processes. Our principal result is a characterization of transmission loss of this bound mode. Loosely speaking, the bound mode transmission coefficient has the following form:

$$| t(l) | \approx \exp[-(\gamma + \kappa + \rho)l]$$

where γ, κ and ρ are positive real constants arising from different physical mechanisms. The parameter γ represents attenuation caused by localization, i.e. the bound-mode-to-bound-mode multiple scattering induced by the random homogeneities. The parameter κ arises because of radiation loss, energy lost by the bound mode to radiation. The result is what one expects on physical grounds. Prior to the asymptotic limit being taken, the original system accounts for scattering from the bound mode to the radiation modes and vice versa. In the limit, however, the coupling has an irreversible quality; energy is lost by the bound mode to the radiation spectrum. An interesting aspect of the result obtained is the fact that both propagating and evanescent radiation modes participate in this loss mechanism and contribute to κ. Often, evanescent radiation modes are neglected on the basis that they do not propagate along the fiber axis. They do propagate transversely to this axis, however. Thus they can (and do) transport energy from the bound mode to the radiation field. The third parameter, ρ, accounts for intrinsic dissipation in the fiber. Explicit expressions have been derived for each of these parameters. They involve power spectral evaluations of stochastic processes arising from the coupling coefficients.

The analysis has two positive benefits. First, it provides a quantitative assessment of the losses experienced by commercial grade single mode (e-core) optical fibers. Second, and perhaps more importantly, it provides an insight and quantitative understanding of the coupling to radiation. As a consequence of this study, and subsequent discussions with the Fiber Optics group at Virginia Tech, the following two potential applications are now being considered:

(I) Polarizer:

Circular core optical fibers support two degenerate dominant modes (the HE_{11} modes); these modes are orthogonally polarized. A polarizer is a device that selectively suppresses one of these two polarizations, leading to a linearly polarized signal. Various schemes have been considered; one such scheme involves a periodic fiber crimping, wherein the introduction of microbends having a specific geometry scatters one of the polarizations into radiation more than the other. We are now considering an alternate scheme wherein the fiber core would be doped with anisotropic scatterers over a prescribed length. The net effect would be to produce an anisotropic propagation environment within the core, with one HE_{11} polarization experiencing more scattering to radiation than the other.

(II) Transverse Illuminator:

In this application, the basic mechanism being studied is again the scattering to radiation by imperfections or scatterers introduced in the fiber core. The objective here would be to create desired lateral illumination along certain fiber segments where the cladding jacket has obviously been removed.

In both of these applications, then, scattering to radiation is deliberately introduced to achieve desired objectives.

A first draft of the research completed thus far has been written. It will be submitted for publication once some rewriting and supporting calculations have been completed.

Personnel Supported:

The following Mathematics graduate students received some support during the grant period:
(a) Elizabeth Bonawitz
(b) Michael Puls
(c) Jeong H. Kim
(d) Lewis Buterakos

Jeong H. Kim was awarded the Ph.D. degree in Mathematics by Virginia Tech (9/94). His thesis is entitled "Stochastic Turning Point Problem". He has developed the following two publications from his thesis material:

"Stochastic turning-point problem for a one-dimensional refractive random multilayer", J. H. Kim, SIAM J. Appl. Math, to appear (6/96)

"A uniform diffusion limit for random wave propagation with turning point", J. H. Kim, J. Math. Phys., to appear (2/96).

The problem considered by Kim was acoustic wave propagation in a randomly layered half space wherein zero mean sound speed fluctuations are superposed upon an upwardly refracting deterministic profile. Oblique monochromatic plane wave excitation was assumed. In the absence of the layering (and in the scaling regime considered), one would be confronted with a classical turning point problem; one could obtain the solution by asymptotically connecting the two outer solutions (below and above the turning point) through a transition layer. The random layering complicates the problem. In particular, the effect of the layering proves in some sense to be greatest when the rays defined by the deterministic profile are nearly horizontal. Kim's work involved adapting and extending known limit theorems to deal with this problem.

Four studies involving the propagation of waves in random media were conducted. In the first of these, monochromatic elastic plane wave propagation within a randomly layered slab was considered. The issues of mode conversion upon reflection and localization were addressed. Mode conversion refers to the conversion of compressional to shear waves and vice versa by multiple scattering within the layering; localiation is the phenomenon wherein the random layering suppresses transmission of energy through the slab. Explicit characterizations of both phenomena were achieved. The second study addressed the issue of inversion. Radiation from an acoustic point source is incident upon a randomly layered acoustic half space characterized by a deterministic refracting profile. The feasibility of reconstructing this refracting profile using the data recorded by multiple surface sensors from a single random realization was shown. The third study dealt with the issue of layering perturbations. A locally layered acoustic medium was considered, i.e. a medium in which random layering is subjected to controlled deformations or undulations.The basic question considered was the robustness of layering theory, do localization and other phenomena associated with random layering theory persist when the plane layers are perturbed. The results obtained establish that many of these properties are in fact preserved. The fourth study dealt with propagation loss in a single mode optical fiber. Of interest was the quantitative characterization of transmission loss, which arises from actual dissipation, localization and radiation loss.